

Gaugino Condensates and Fluxes in $N = 1$ Effective Superpotentials*

J.-P. Derendinger,¹ C. Kounnas² and P.M. Petropoulos³

¹ Physics Institute, Neuchâtel University, A.-L. Breguet 1,
 CH-2000 Neuchâtel, Switzerland

² Laboratoire de Physique Théorique, Ecole Normale Supérieure,[†]
 24 rue Lhomond, F-75231 Paris Cedex 05, France

³ Centre de Physique Théorique, Ecole Polytechnique,[◇]
 F-91128 Palaiseau, France

Abstract

In the framework of orbifold compactifications of heterotic and type II orientifolds, we study effective $N = 1$ supergravity potentials arising from fluxes and gaugino condensates. These string solutions display a broad phenomenology which we analyze using the method of $N = 4$ supergravity gaugings. We give examples in type II and heterotic compactifications of combined fluxes and condensates leading to vacua with naturally small supersymmetry breaking scale controlled by the condensate, cases where the supersymmetry breaking scale is specified by the fluxes even in the presence of a condensate and also examples where fluxes and condensates conspire to preserve supersymmetry.

* Research partially supported by the EU under the contracts MEXT-CT-2003-509661, MRTN-CT-2004-005104 and MRTN-CT-2004-503369.

[†] Unité mixte du CNRS et de l'Ecole Normale Supérieure, UMR 8549.

[◇] Unité mixte du CNRS et de l'Ecole Polytechnique, UMR 7644.

1 Introduction

Superstring constructions provide a plethora of four-dimensional vacua with exact or spontaneously broken supersymmetries. Those with chiral-fermion families have $N = 1$ or $N = 0$ supersymmetry in four dimensions. Such models can be built in the framework of heterotic compactifications or type II orientifolds with branes and fluxes.

In the case of vacua with $N = 1$ supersymmetry, it is expected that nonperturbative phenomena break the leftover supersymmetry. These nonperturbative effects are controlled by gaugino condensation occurring in the infrared regime of strongly coupled gauge sectors [1, 2]. Both approaches predict modifications in the superpotential of the effective supergravity theory. The leading terms are in general of the form

$$W_{\text{nonpert}} = \mu^3 \exp\left(-\frac{24\pi^2 Z}{b_0}\right), \quad (1.1)$$

where b_0 is a one-loop beta-function coefficient, μ a scale at which the Wilson coupling $g^2(\mu)$ is defined and Z a modulus such that $\text{Re } Z = g^{-2}(\mu)$. The expectation value of the nonperturbative superpotential defines the renormalization-group-invariant transmutation scale Λ of the confining gauge sector in which gauginos condense, $\langle W_{\text{nonpert.}} \rangle = \Lambda^3$. In a given string compactification, physical quantities are functions of moduli fields. In particular, *the scale μ itself is in general a modulus-dependent quantity*. Typically, the exponent in the nonperturbative superpotential (1.1) is a number of order ten or more and N -instanton corrections ($N > 1$) are exponentially suppressed. But several condensates could form and the nonperturbative superpotential could include several similar terms involving various moduli.

Which modulus Z appears in Eq. (1.1) depends on the underlying string (or M-) theory. In the heterotic string, it is identified with the dilaton field S [2, 3]. In type II orientifolds, Z is a well-defined combination of U and S in IIA theories and of T and S in IIB (or F-theory) compactifications [4, 5, 6].

These nonperturbative contributions coexist, in the effective superpotential, with the perturbative moduli-dependent terms produced by compactifications of heterotic [2, 7], type IIA [8] and type IIB [9] compactifications with various types of fluxes. Recently, a method has been developed to determine unambiguously the Kähler potential and the superpotential of the effective supergravity in terms of *all* fluxes present in the fundamental theory [5].

This allows for an exhaustive analysis, in particular for the type IIA where no systematic Calabi–Yau approach exists due to the presence of torsion induced by geometrical fluxes (“*generalized*” Calabi–Yau) [10]. This includes a large class of (freely acting) orbifolds or fermionic string models [11]. The principle of the method is that the $N = 1$ theory at hand is strongly constrained by the universality properties of the underlying $N \geq 4$ extended supergravity, inherited from the string fundamental theory before the orbifold projection. The only available tool for generating a non-trivial potential in $N \geq 4$ supergravity theories is the *gauging procedure* [12], ultimately related to the nature and strength of the fluxes present in the theory. The gauging procedure determines completely the $N = 1$ superpotential, at least for the bulk fields. Those are relevant for the determination of the vacuum structure, before considering the usual perturbative corrections in the observable sector, originating from the renormalization of the softly broken $N = 1$ supersymmetry.

Although the inclusion of the nonperturbative corrections in the flux-induced superpotential has been proposed by several authors in early and recent literature [13], the conclusions have been either controversial or incomplete, mainly due to the pathological behaviour of the vacuum, like for instance:

- (i) runaway behaviour of the moduli Z ,
- (ii) destabilization of the no-scale structure/positivity of the effective potential,
- (iii) undesired transitions to anti-de Sitter vacua,
- (iv) fine-tuning problem associated with the quantization of the flux coefficients.

Having the generic and unambiguous structure of the effective superpotential in the presence of fluxes, we will reexamine these issues and show that these pathologies of the vacuum can be avoided with a suitable choice of fluxes in heterotic, IIA and IIB $N = 1$ effective supergravities. As we will see in several examples, the nonperturbative contribution involving the superfield Z *is not always* the source for supersymmetry breaking.

Our paper has mainly two parts. In Sec. 2, we first recall the generation of $N = 1$ superpotentials from fluxes, following Ref. [5]. We then summarize the general framework for describing condensates in an effective Lagrangian approach and describe some of the common caveats of the method. Section 3 is devoted to the study of examples illustrating

several scenarios: (i) situations where supersymmetry breaking is independent from gaugino condensation and (ii) cases where the gaugino condensates induce supersymmetry breaking. These examples fall into three different classes with respect to the scaling properties of the gravitino mass. The difficulties one usually encounters when including the nonperturbative corrections are clearly described. The resolution of these problems is possible provided one realizes that the issues of moduli stabilization, supersymmetry breaking, gaugino condensation and positivity of the potential, although related, must be treated *separately*. Numerous examples can be found both in type II and heterotic where supersymmetry breaking is driven by fluxes, the condensate playing a subdominant role. One can also exhibit generic type II models where the gaugino condensate itself is responsible for the supersymmetry breaking. Similar realizations exist in heterotic string provided the fluxes are chosen to be large, with ratios of order one though. A summary and some comments are given in the last section.

2 Superpotentials, gaugings and condensates

2.1 Flux dependence of the superpotential

We will confine our discussion to the $Z_2 \times Z_2$ orbifold compactification of heterotic strings, or IIA and IIB orientifolds. This compactification setup leads to seven main moduli (including the string dilaton) and $N = 1$ supersymmetry. The effective supergravity is the $Z_2 \times Z_2$ projection of the $N = 4$ theory which would describe the sixteen-supercharge ten-dimensional theory (on a simple six-torus). Fluxes are introduced by gauging the $N = 4$ supergravity. As shown in Ref. [5], the gauging allows the introduction of NS-NS, R-R fluxes as well as geometric fluxes [5, 14] corresponding to non-trivial internal spin connections.

Let us restrict ourselves to the main moduli, namely the string coupling, the six geometrical moduli fields and their superpartners. Their precise spelling depends on the theory under consideration. For heterotic strings on $T^6/Z^2 \times Z^2$, these are the dilaton-axion superfield S , the volume moduli T_A and the complex-structure moduli U_A , $A = 1, 2, 3$. The index A refers to the three complex planes left invariant by $Z_2 \times Z_2$. The $N = 1$ supersymmetric complexification for these fields is defined naturally in terms of the geometrical moduli G_{ij} (nine fields), the string dilaton Φ and the components B_{ij} (three fields) and $B_{\mu\nu} \sim a$ of the

antisymmetric tensor. Explicitly, the metric tensor restricted to the plane A is

$$(G_{ij})_A = \frac{t_A}{u_A} \begin{pmatrix} u_A^2 + \nu_A^2 & \nu_A \\ \nu_A & 1 \end{pmatrix}, \quad (2.1)$$

with

$$T_A = t_A + i(B_{ij})_A, \quad U_A = u_A + i\nu_A \quad (2.2)$$

and

$$e^{-2\Phi} = s(t_1 t_2 t_3)^{-1}, \quad S = s + ia. \quad (2.3)$$

The Weyl rescaling to the four-dimensional Einstein frame is $G_{ij} = s^{-1} \tilde{G}_{ij}$. The $Z_2 \times Z_2$ projection of the $N = 4$ theory leads to the scalar Kähler manifold

$$M = \left[\frac{SU(1,1)}{U(1)} \right]_S \times \prod_{A=1}^3 \left[\frac{SU(1,1)}{U(1)} \right]_{T_A} \times \prod_{A=1}^3 \left[\frac{SU(1,1)}{U(1)} \right]_{U_A}, \quad (2.4)$$

with Kähler potential (in the usual string parameterization)

$$K = -\log(S + \bar{S}) - \sum_{A=1}^3 \log(T_A + \bar{T}_A) (U_A + \bar{U}_A). \quad (2.5)$$

Coupling the seven moduli superfields to further multiplets Z_A^I , including gauge-charged states, modifies however the Kähler potential of the $N = 1$ theory, to become

$$K = -\log(S + \bar{S}) - \sum_{A=1}^3 \log \left[(T_A + \bar{T}_A) (U_A + \bar{U}_A) - \sum_{I=1}^{n_A} (Z_A^I + \bar{Z}_A^I)^2 \right]. \quad (2.6)$$

The superpotential, as explained in Ref. [5], is determined from the gravitino mass term of the $N = 4$ gauged supergravity, after the orbifold and orientifold projections:

$$e^{K/2} W = \frac{1}{6} \epsilon^{ABC} \left[(\phi_0 - \phi_1) f_{IJK} + (\phi_0 + \phi_1) \bar{f}_{IJK} \right] \Phi_A^I \Phi_B^J \Phi_C^K, \quad (2.7)$$

where the antisymmetric numbers f_{IJK} and \bar{f}_{IJK} are the structure constants of the $N = 4$ gauged algebra multiplied by the $SO(6, n)$ metric and $SU(1, 1)$ S -duality phases [12]¹. The constrained fields

$$\phi_0, \phi_1 \quad \text{and} \quad \Phi_A^I = \{\sigma_A^1, \sigma_A^2; \rho_A^1, \rho_A^2, \chi_A^I\} \quad (2.8)$$

¹The indices I, J, K label the vector representation of the $SO(6, n)$ global symmetry of the (ungauged) $N = 4$ theory and η_{IJ} is the invariant metric for this representation. Then, for instance, $f_{IJK} = f_{IJ}{}^L \eta_{LK}$, where $f_{IJ}{}^L$ is a structure constant.

define the scalar manifold as the solution of the following constraint equations:

$$\begin{aligned}
\frac{1}{2} &= |\phi_0|^2 - |\phi_1|^2, \\
\frac{1}{2} &= |\sigma_A^1|^2 + |\sigma_A^2|^2 - |\rho_A^1|^2 - |\rho_A^2|^2 - \sum_I |\chi_A^I|^2, \\
0 &= (\sigma_A^1)^2 + (\sigma_A^2)^2 - (\rho_A^1)^2 - (\rho_A^2)^2 - \sum_I (\chi_A^I)^2, \quad (A = 1, 2, 3).
\end{aligned} \tag{2.9}$$

Since our goal is to work with a fixed set of well-defined moduli fields (S, T_A, U_A) , and to study various classes of gaugings induced by the fluxes, it is necessary to solve the above constraints in terms of the fields (S, T_A, U_A, Z_A^I) :

- S manifold $[SU(1, 1)/U(1)]_S$:

$$\phi_0 - \phi_1 = \frac{1}{(S + \bar{S})^{1/2}}, \quad \phi_0 + \phi_1 = \frac{S}{(S + \bar{S})^{1/2}}; \tag{2.10}$$

- T_A, U_A, Z_A^I manifold $[SO(2, 2 + n_A)/SO(2) \times SO(2 + n_A)]_{T_A, U_A, Z_A^I}$:

$$\begin{aligned}
\sigma_A^1 &= \frac{1}{2Y_A^{1/2}} \left(1 + T_A U_A - (Z_A^I)^2 \right), & \sigma_A^2 &= \frac{i}{2Y_A^{1/2}} (T_A + U_A), \\
\rho_A^1 &= \frac{1}{2Y_A^{1/2}} \left(1 - T_A U_A + (Z_A^I)^2 \right), & \rho_A^2 &= \frac{i}{2Y_A^{1/2}} (T_A - U_A), \\
\chi_A^I &= \frac{i}{Y_A^{1/2}} Z_A^I,
\end{aligned} \tag{2.11}$$

$$\text{where } Y_A = (T_A + \bar{T}_A)(U_A + \bar{U}_A) - \sum_I (Z_A^I + \bar{Z}_A^I)^2.$$

The above equations allow to rewrite the scalar potential and the gravitino mass term as functions of the $N = 1$ complex scalars, the perturbative f_{IJK} -term as well as the nonperturbative S -dual term, $S\bar{f}_{IJK}$. The Kähler potential and the superpotential can then be obtained by separating the holomorphic part in the $N = 1$ gravitino mass term, using the relation $m_{3/2} = e^{K/2}W$. The result is the Kähler potential given above in Eq. (2.6).

It is important to realize that even if the scalar manifolds for type IIA or IIB orientifold compactifications are the same as for heterotic strings, the identification of the complex scalar fields present in the seven chiral multiplets S, T_A, U_A changes. This is obvious for the imaginary parts since tensor fields change with the ten-dimensional string theory. For the real parts, the relation between the seven main moduli s, t_A and u_A , as defined from

the metric and the dilaton in Eqs. (2.1–2.3), and the real parts of S , T_A and U_A is a (non-supersymmetric) field redefinition which leaves invariant the kinetic terms². The appropriate field redefinition is dictated by the dilaton rescaling and the orientifold projection. Consider for instance the type IIA orientifold with D_6 -branes. While the superfields T_A are the same as for heterotic strings, S and U_A are now defined by

$$\begin{aligned} S &= s' + iA_{6810}, \\ U_1 &= u'_1 - iA_{679}, \quad U_2 = u'_2 - iA_{589}, \quad U_3 = u'_3 - iA_{5710}, \end{aligned} \quad (2.12)$$

in terms of the $(Z_2 \times Z_2)$ -invariant components of the R-R three-form A_{ijk} ($i, j, k = 5, \dots, 10$).

The relations

$$s' = \sqrt{\frac{s}{u_1 u_2 u_3}}, \quad u'_1 = \sqrt{\frac{s u_2 u_3}{u_1}}, \quad u'_2 = \sqrt{\frac{s u_1 u_3}{u_2}}, \quad u'_3 = \sqrt{\frac{s u_1 u_2}{u_3}} \quad (2.13)$$

define then the real parts in terms of the geometric moduli introduced in Eqs. (2.1–2.3).

We can illustrate the above in the T^6/Z_2 type IIB orientifold with a $Z_2 \times Z_2$ orbifold projection. The Z_2 inverts all six internal coordinates and this is compatible with the presence of D_3 -branes. A field redefinition similar to Eqs. (2.13) but involving s and t_A is again necessary. The NS-NS and R-R three-forms generate fluxes compatible with the $Z_2 \times Z_2$ setup. These fluxes lead to the following superpotential terms:

$$\begin{aligned} \text{R-R three-form:} \quad & 1 && (\text{from } F_{579}), \\ & i(U_1 + U_2 + U_3) && (\text{from } F_{679} = F_{589} = F_{5710}), \\ & -(U_1 U_2 + U_2 U_3 + U_3 U_1) && (\text{from } F_{689} = F_{5810} = F_{6710}), \\ & -iU_1 U_2 U_3 && (\text{from } F_{6810}), \\ \text{NS-NS three-form:} \quad & iS && (\text{from } H_{579}), \\ & -S(U_1 + U_2 + U_3) && (\text{from } H_{679} = H_{589} = H_{5710}), \\ & -iS(U_1 U_2 + U_2 U_3 + U_3 U_1) && (\text{from } H_{689} = H_{5810} = H_{6710}), \\ & SU_1 U_2 U_3 && (\text{from } H_{6810}). \end{aligned} \quad (2.14)$$

More fluxes are allowed in type IIA or heterotic models, where also geometric fluxes³ are permitted by the orbifold projection. These fluxes would induce other (possibly T_A -dependent) terms in the superpotential. Those are classified in [5], and we will use these terms in the examples presented in Sec. 3.

²The kinetic terms for the main moduli t_A and u_A arise from the compactification of the Einstein term and are thus universal.

³Equivalent to background values of spin-connection components.

2.2 Effective field theory description of condensates

The generation of a nonperturbative superpotential due to gaugino condensates follows schematically from the following argument. In $N = 1$ supersymmetric field theories, the coupling of gauge fields to chiral multiplets involves a holomorphic function,

$$\frac{1}{4} \int d^2\theta f(\phi^i, M^a) WW + \text{h.c.},$$

where ϕ^i denotes the charged matter chiral superfields and M^a the (gauge-neutral) moduli fields. In the Wilson Lagrangian derived from a superstring theory, the function f receives contributions from several sources, including renormalization-group (one-loop) running and field-dependent threshold corrections due to charged matter and moduli fields.

To obtain the effective action for gaugino condensates, one first introduces a composite classical chiral superfield $U \simeq \langle WW \rangle$. The dynamics of U is then (mostly) dictated by anomaly-matching. Gauge kinetic terms are replaced by an F -density Lagrangian with in general three contributions:

$$\frac{1}{4} \int d^2\theta \left[f_{\text{eff}}(M^a, \phi^i) U + w_{VY}(U) + w_K(M^a, \phi^i) U \right] + \text{h.c.} \quad (2.15)$$

Suppose that the (string tree-level) gauge coupling field is related to the modulus S , $\text{Re } S = g_W^{-2}$. The holomorphic function f_{eff} is then⁴

$$Z \equiv f_{\text{eff}} = S + f_{\text{thr}}(M^a, \phi^i), \quad (2.16)$$

where f_{thr} would describe moduli-dependent threshold corrections. The modulus Z appearing in the introduction is actually f_{eff} . The leading behaviour of typical string theory threshold corrections is linear in some combination of moduli fields [15], or logarithmic [16]. Field theory thresholds, which in general have a logarithmic behaviour, are actually described by w_K , as discussed below. The second contribution in expression (2.15) is the Veneziano–Yankielowicz superpotential matching the anomaly of the superconformal chiral $U(1)$ [17]:

$$w_{VY}(U) = \frac{b_0}{24\pi^2} U \left[\ln \left(\frac{U}{\mu^3} \right) - 1 \right], \quad (2.17)$$

where $b_0 = 3C(G) - T(R)$ is the one-loop beta-function coefficient and μ is the scale at which the Wilson action has been defined⁵, $\text{Re } S = g_W^{-2}(\mu)$. Finally, Konishi anomalies [18] related

⁴Up to irrelevant constants if the gauge group is not simple.

⁵The scale μ is the ultra-violet cutoff of the wilsonian action.

to chiral $U(1)$ (non R -) symmetries acting on each charged matter representation generate the last term, which more precisely reads

$$\frac{1}{4} \sum_I \frac{b_I}{24\pi^2} \int d^2\theta U \ln \left(\frac{\mathcal{P}_I(\phi^i, M^a)}{U^{p_I}} \right) + \text{h.c.}$$

It does not depend on the Wilson scale μ^6 . The sum is over the irreducible representations of the chiral superfields and b_I is a positive coefficient dictated by anomaly-matching. This term can also be written as

$$\frac{1}{96\pi^2} \int d^2\theta U \ln \left(\frac{\mathcal{P}(\phi^i, M^a)}{U^p} \right) + \text{h.c.}, \quad \mathcal{P}(\phi^i, M^a) = \prod_I \mathcal{P}_I(\phi^i, M^a)^{b_I},$$

with $p = \sum_I b_I p_I$. The quantity \mathcal{P} is an analytic gauge invariant polynomial which characterizes matter condensates along the D -flat directions of the scalar potential and the exponent p is such that the argument of the logarithm is dimensionless [20, 21].

The nonperturbative superpotential is then obtained by eliminating U in the F -density, with the result (if $p \neq b_0$)

$$W_{\text{nonpert}} = W_{\text{classical}}(M^a, \phi^i) + \mu^3 e^{\frac{p}{b_0-p}} \left(\frac{\mathcal{P}}{\mu^{3p}} \right)^{\frac{1}{p-b_0}} \exp \left[-\frac{24\pi^2}{b_0-p} (S + f_{\text{thr}}) \right]. \quad (2.18)$$

For a symmetry breaking $G \rightarrow H$, the positive number p is given by $b_0|_G - b_0|_H = 2T(R_{\text{Goldstone}})$, where $R_{\text{Goldstone}}$ is the representation of the coset G/H . Notice that the term matching Konishi anomalies modifies the Veneziano–Yankielowicz term. This modification is precisely required by the threshold induced when chiral fields break gauge symmetries⁷.

The terms matching Konishi anomalies and inducing field theory thresholds produce then an effective, field-dependent scale

$$\mu_{\text{eff}}^3(M^a, \phi^i) = \mu^3 e^{\frac{p}{b_0-p}} \left(\frac{\mathcal{P}}{\mu^{3p}} \right)^{\frac{1}{p-b_0}} \quad (2.19)$$

in front of the usual superpotential with exponential behaviour in the gauge coupling field S supplemented by string threshold contributions.

The particular case $p = b_0$ would eliminate completely the $U \ln U$ contribution in the effective superpotential. As an example, consider $N = 2$ super-Yang–Mills theory. In terms

⁶It is an “infrared” contribution describing in the wilsonnian action the physics of spontaneous symmetry breakings induced by chiral fields.

⁷These are field theory thresholds induced in the range of validity of the Wilson Lagrangian, at scales lower than the ultra-violet cutoff μ .

of $N = 1$ superfields, the theory includes a single adjoint chiral superfield which generically breaks the gauge group to its abelian Cartan subgroup. Using then $\mathcal{P}_I(\phi) = \text{Tr}(\phi^2)$, anomaly matching requires $p = 2C(G) = b_0$ (and $b_I = 3/2$). As a result, the effective superpotential is simply

$$\frac{1}{4} U \left[S + \frac{b_0}{16\pi^2} \ln \left(\frac{\text{Tr}(\phi^2)}{\mu^2} \right) \right]$$

and eliminating U cancels the nonperturbative superpotential with however

$$\text{Tr}(\phi^2) = \mu^2 e^{-16\pi^2 S/b_0}. \quad (2.20)$$

In a general $N = 1$ case with $p = b_0$,

$$W_{\text{nonpert}} = W_{\text{classical}}(M^a, \phi^i) \quad (2.21)$$

and the scale of the invariant \mathcal{P} is fixed by the equation

$$\mathcal{P}(\phi^i, M^a) = \mu^{3b_0} e^{-24\pi^2 [S + f_{\text{thr}}(M^a, \phi^i)]}. \quad (2.22)$$

Even if the condensate contribution to the nonperturbative superpotential is cancelled, the matter fields show an exponential behaviour in the gauge coupling field. This behaviour will explicitly reappear in the coupling, via the classical superpotential, of matter fields ϕ^i to gauge singlet fields (like moduli fields) or to “spectator” charged matter superfields. Again in the $N = 2$ example, hypermultiplets couple to the gauge multiplet via the (classical) superpotential term $\sqrt{2}H^c\phi H$. Expression (2.20) generates then a mass term of the form

$$\sim \mu e^{-8\pi^2 S/b_0} H^c T H, \quad (2.23)$$

with some constant matrix T .

2.3 The condensate scale, supersymmetry breaking and the runaway problem

Let us now review in a simple situation the role of the condensate superpotential in defining the vacuum structure of the effective supergravity, illustrating the fine-tuning problem alluded to in the introduction. Consider the following simplified superpotential:

$$W = a + w(S), \quad w(S) = \mu^3 e^{-S} \quad (2.24)$$

and we have for convenience rescaled S according to

$$\frac{24\pi^2 S}{b_0} \longrightarrow S.$$

This rescaling leaves the corresponding kinetic terms unchanged and multiplies the scalar potential by an overall factor. The quantity a is in general T_A - and U_A -dependent. It includes the perturbative contributions to the superpotential induced by fluxes. In a heterotic compactification with vanishing geometrical fluxes however, only the U_A -dependence generated by the heterotic three-form H_3 does survive. As already stated, the scale μ may depend on moduli U_A or T_A , but a does not depend on S in heterotic compactifications.

Due to the $SU(1,1)$ structure of the Kähler manifold, the scalar potential considerably simplifies and takes the following suggestive form:

$$e^{-K}V = \sum_i \left| W - W_i(Z_i + \bar{Z}_i) \right|^2 - 3|W|^2, \quad (2.25)$$

where $\{Z_i\} \equiv \{S, T_A, U_A\}$ and $W_i = \partial_{Z_i} W$.

Consider now situations where the geometrical fluxes are indeed absent and the superpotential is T_A -independent. The scalar potential becomes:

$$e^{-K}V = \sum_{\{Z_i\} \equiv \{S, U_A\}} \left| W - W_i(Z_i + \bar{Z}_i) \right|^2. \quad (2.26)$$

We are led to a *no-scale model* [22], with a semi-positive-definite potential and flat directions $\{T_A\}$. The U_A moduli are generically fixed by their minimization conditions and a and μ^3 in Eqs. (2.24) are effectively constant. The remaining minimization condition for the S field,

$$a + (S + \bar{S} + 1) w(S) = 0, \quad (2.27)$$

determines the value of S . Supersymmetry is broken in the T_A -directions, in Minkowski space. Equation (2.27) shows that an exponentially small value of $w(S)$ necessarily implies

$$|a| \ll 1, \quad (2.28)$$

a fine-tuning condition on the perturbative fluxes.

This is a potential problem in cases with U_A -independent a , like for instance in the Z_3 orbifold where the would-be U_A moduli are frozen to their self-dual values by the orbifold

point group action and a is then given by the (constant) H_3 -form flux. Because of flux quantization, the perturbative superpotential a is therefore of $\mathcal{O}(1)$ or zero and a non-zero a is in contradiction with the minimum condition. A vanishing a is also problematic since it leads to a runaway potential:

$$V = \frac{(1 + S + \bar{S})^2}{(S + \bar{S}) \prod_A (T_A + \bar{T}_A)} |\mu|^6 e^{-(S + \bar{S})}. \quad (2.29)$$

Various attempts have been proposed in the past for improving this situation (see for instance [19]), including multiple gauge group condensations (without fluxes) which however do not help in removing the fine-tuning problem with a non-zero a .

The above conclusions are drastically modified with a modulus-dependent perturbative superpotential, as induced by fluxes. For instance, type IIB models with stable moduli S and U_A have automatically a no-scale structure with all T_A moduli being flat directions. For example, this can be realized with

$$\begin{aligned} W = & A [1 + U_1 U_2 + U_2 U_3 + U_3 U_1 + \gamma S (U_1 + U_2 + U_3 + U_1 U_2 U_3)] \\ & + iB [U_1 + U_2 + U_3 + U_1 U_2 U_3 + \gamma S (1 + U_1 U_2 + U_2 U_3 + U_3 U_1)], \end{aligned} \quad (2.30)$$

where A, B and $\gamma A, \gamma B$ are respectively proportional to R-R and NS-NS flux numbers. The absence of any T_A dependence in W is responsible for the appearance of the no-scale structure with semi-positive-definite scalar potential. This superpotential fixes the moduli $(U_A, \gamma S)$ to unity and supersymmetry is broken in flat space with gravitino mass $m_{3/2}^2 \propto (A^2 + B^2) / \prod_A (T_A + \bar{T}_A)$. A plethora of examples exists with similar properties, or with unbroken supersymmetry whenever W vanishes.

We now observe that in this example, one expects nonperturbative superpotential contributions proportional to $\exp -\alpha_A T_A$ arising from condensates that live on D_7 -branes⁸. To be concrete, add to W in Eq. (2.30) a nonperturbative contribution $\exp[-\alpha(T_1 + T_2 + T_3)] = \exp(-3\alpha T)$ (all T_A equal). It is clear from the above analysis that this addition spoils *arbitrarily* the no-scale structure and destabilizes the Minkowski vacuum: the moduli T gets stabilized but the potential becomes negative, $V = -3m_{3/2}^2$, as required by unbroken supersymmetry in anti-de Sitter space. Notice that positivity would have been lost as well, had $w(T)$ been a function of a single T_A modulus. In such a case however, the potential does not

⁸While contributions proportional to $e^{-\alpha S}$ would arise from D_3 -branes.

have a critical point and the fundamental state of the theory is described by domain-wall-like solutions with 1/2 supersymmetry.

In this example, the vacuum structure of the theory is clearly only understood from the analysis of the combined perturbative and nonperturbative contributions. It makes little sense to focus first on stabilizing moduli from the flux superpotential only which is actually “too stable” to lead to relevant phenomenology once the condensate contributions are added.

3 Gaugino condensation, stabilization of moduli and supersymmetry breaking

Nonperturbative phenomena do not necessarily break supersymmetry, independently of their effect on the stabilization of the moduli and on the positivity of the potential. To illustrate these statements, we will proceed by analyzing some examples, insisting on the role of the various terms that can appear in the superpotential.

3.1 Some properties of the scalar potential stationary points

The analysis of the non-positive scalar potential depending on seven complex fields is a difficult task when supersymmetry breaks. We will use in this section some properties of the potential which follow from our particular forms of the Kähler potential and of the superpotential.

In a general supergravity theory with Kähler potential $K = -\sum_j \ln(Z_j + \bar{Z}_j)$, supersymmetry is spontaneously broken if the equations

$$F_j \equiv W - (Z_j + \bar{Z}_j)W_j = 0 \quad (3.1)$$

cannot be solved for all scalar fields Z_j (and with $\text{Re } Z_j > 0$). In this case, our first goal is to find stationary points where supersymmetry breaks in Minkowski space:

$$\langle V \rangle = 0, \quad \langle W \rangle \neq 0. \quad (3.2)$$

A stationary point of the scalar potential is a solution of the equation $\partial_j V = 0$, $\forall j$, which explicitly reads:

$$0 = e^{-K} V K_j - \bar{W}_j F_j - 3W_j \bar{W} + \sum_{i \text{ with } i \neq j} \left[W_j - (Z_i + \bar{Z}_i)W_{ij} \right] \bar{F}_i - (Z_j + \bar{Z}_j)W_{jj}\bar{F}_j \quad (3.3)$$

for each scalar field Z_j . The first term vanishes at a Minkowski point and the second derivative W_{jj} only exists for the modulus appearing in the exponential gaugino condensate term.

Suppose now that $\langle W_j \rangle = 0$ for a certain modulus. Then $\langle F_j \rangle = \langle W \rangle \neq 0$ and the contribution of the field to $\langle V \rangle$ cancels one unit of the negative term $-3\langle W\overline{W} \rangle$. We then consider the case where the scalar fields split in two categories, with either $\langle W_j \rangle = 0$ and $\langle F_j \rangle = \langle W \rangle \neq 0$, or with $\langle F_j \rangle = 0$. Supersymmetry breaking is controlled by the first category only. The Minkowski condition, $\langle V \rangle = 0$ implies then that this category contains precisely three fields.

For a field such that $\langle W_j \rangle = 0$, the stationarity condition (3.3) simplifies to⁹

$$0 = \sum_{i \text{ with } i \neq j} (Z_i + \overline{Z}_i) W_{ij} \overline{F}_i. \quad (3.4)$$

Only directions with broken supersymmetry contribute to the summation, since for the others $\langle F_i \rangle = 0$. For those which contribute, $\langle F_i \rangle = \langle W \rangle$ and therefore Eq. (3.4) reduces to

$$0 = \sum_{i \text{ with } i \neq j} W_{ij} \operatorname{Re} Z_i, \quad (3.5)$$

where the summation includes only fields Z_i such that $\langle W_i \rangle = 0$. Notice that this equation is trivially satisfied whenever fields such that $\langle W_j \rangle = 0$ couple in the superpotential only to directions where supersymmetry does not break. In this case, W_{ij} in Eq. (3.5) vanishes since $W_{ij} \neq 0 \iff \langle F_i \rangle = 0$.

Consider now a modulus Z_j for which $\langle F_j \rangle = 0$. Equation (3.3) for this field becomes

$$0 = -3W_j \overline{W} + \sum_{i \text{ with } i \neq j} \left[W_j - (Z_i + \overline{Z}_i) W_{ij} \right] \overline{F}_i. \quad (3.6)$$

The sum runs over fields for which $\langle F_i \rangle \neq 0$. For those, $\langle W_i \rangle = 0$ and therefore $\langle F_i \rangle = \langle W \rangle$. Since there are three such fields, Eq. (3.6) is equivalent to

$$0 = \sum_{i \text{ with } i \neq j} W_{ij} \operatorname{Re} Z_i, \quad (3.7)$$

with a summation over all fields Z_i such that $\langle W_i \rangle = 0$. Hence, both conditions can be summarized by the seven conditions

$$0 = \sum_{i \text{ with } i \neq j} \langle W_{ij} \operatorname{Re} Z_i \rangle, \quad (3.8)$$

⁹This assumption will not be used for the modulus controlling the gaugino condensate.

with a summation restricted over moduli which break supersymmetry.

In the examples studied below, the structure of the superpotential gives indeed a partition between moduli for which $\langle W_j \rangle = 0$ and supersymmetry breaks, $\langle F_j \rangle \neq 0$, and moduli for which supersymmetry does not break $\langle F_j \rangle = 0$. For the first class, the stationarity of the potential reduces to the simple equation $W_j = 0$ while for the second class, it reduces to Eq. (3.7).

3.2 Supersymmetry breaking independent of the gaugino condensation

Let us start with the case where the gaugino condensate does not break supersymmetry. The introduction of fluxes can indeed stabilize some of the moduli in flat space without inducing supersymmetry breaking. To be concrete, let us consider a superpotential with “supersymmetric mass terms” only:

$$\begin{aligned} W_{\text{susy}} = & A(U_1 - U_2)(T_1 - T_2) + B(U_1 + U_2 - 2U_3)(T_1 + T_2 - 2T_3) \\ & + C(U_1 - U_2)(T_1 + T_2 - 2T_3) + D(T_1 - T_2)(U_1 + U_2 - 2U_3). \end{aligned} \quad (3.9)$$

This superpotential is created by geometrical fluxes, either in heterotic or in type IIA. It can be directly generated at the string level using freely acting orbifold constructions. It selects four (complex) directions in the seven-dimensional space of the moduli fields and minimizing the potential tends to cancel the fields in these four directions. Supersymmetry is not broken and cancellation of auxiliary fields¹⁰ fixes $U_A = U$ and $T_A = T$. There are then flat directions and $W_{\text{susy}} = 0$ at the minimum. Further supersymmetric mass terms are those which mix the T_A and U_A moduli in the minimization conditions:

$$(U_1 - T_1)(U_2 - U_3) \quad \text{or} \quad (U_1 - T_1)(T_2 - T_3). \quad (3.10)$$

The first term is present only in heterotic with geometrical and H_3 fluxes, while the second term appears only in type IIA, when geometrical and F_2 fluxes are switched on. We could also consider

$$(U_1 - T_1)(U_2 - T_2) \quad (3.11)$$

which exists as a four-dimensional $N = 4$ gauging but cannot be obtained from a ten-dimensional field theory by the Scherk–Schwarz mechanism, although it can be realized in freely acting asymmetric heterotic orbifold constructions [11].

¹⁰The conditions $W_i (Z_i + \bar{Z}_i) = W$ for every scalar field Z_i .

In any case, since the superpotential is homogeneous, the normalizations of the T_A and U_A moduli are not fixed and at least one modulus direction remains flat. These supersymmetric mass terms are generic in the sense that they may fix some of the moduli without modifying the minimum of the potential and without breaking supersymmetry. We will use those terms in the following as building blocks, and add nonperturbative contributions.

Our first example is the following:

$$\begin{aligned} W_{\text{susy}} = & A(U_1 - U_2)(T_1 - T_2) + B(U_1 + U_2 - 2U_3)(T_1 + T_2 - 2T_3) \\ & + (T_1 + T_2 - 2T_3)w(S), \end{aligned} \quad (3.12)$$

with $w(S)$ as given in Eq. (2.24). The condensate term $(T_1 + T_2 - 2T_3)w(S)$ can be understood from the general form of the $N = 4$ superpotential, Eq. (2.7). This cubic expression also produces in the superpotential terms proportional to

$$f_{IJK}\epsilon_{ABC}\Phi_A^I Z_B^J Z_C^K$$

where Φ_A^I is linear in the main moduli T_A and U_A while the Z_B^J and Z_C^K are charged under the confining gauge group. As discussed earlier, condensation induces for these charged contributions an exponential dependence on the gauge coupling field of the confining gauge group, say S , as in the last term of superpotential (3.12).

The presence of $w(S)$ leaves $U_1 = U_2 = U$ but we now have $U_3 = U + w(S)/2B$. The above conclusions about supersymmetry remain however unchanged: supersymmetry is unbroken and the gravitino is massless in flat background.

The previous example can be modified by the addition of further flux terms which break supersymmetry. Consider instead, in the heterotic or type IIA

$$W_{\text{total}} = W_{\text{susy}} + W_{\text{break}} \quad (3.13)$$

with W_{susy} as in Eq. (3.12) and

$$W_{\text{break}} = R(T_1 U_1 + T_2 U_2). \quad (3.14)$$

The term W_{break} breaks supersymmetry *even in the absence of* $w(S)$. The scalar potential has a minimum with real T_A , U_A and $T_A = T$, $U_1 = U_2 = U$, $U_3 = U + w(S)/2B$. The potential vanishes along the flat directions S , T and U . The goldstino field is a combination of the fermionic partners of S , T_3 and U_3 . There is an “effective” no-scale structure: since

$W_i = 0$ in these three directions, their corresponding contributions to the potential cancel the gravitational contribution $-3m_{3/2}^2$. Thus supersymmetry is broken in flat space-time with

$$m_{3/2}^2 = \frac{|R|^2}{32 ST_3 U_3}$$

and the presence of the nonperturbative term $w(S)$ only acts as a small perturbation on supersymmetry breaking induced by the modulus-dependent contribution W_{break} . It however explicitly appears in mass terms.

Keeping the same mass terms given in Eq. (3.12) but choosing a different breaking term

$$W_{\text{break}} = R(T_1 S + L U_3 T_3), \quad (3.15)$$

induced by the geometrical fluxes in type IIA, fixes the moduli after minimization to $T_A = T$, $S = L U_3$, $U_3 = U + w(S)/2B$. Now the goldstino direction is a combination of U_1, U_2 and T_2 and in this example, the S -direction does not break supersymmetry.

As another example in the framework of type IIA, keep the same mass terms but replace W_{break} in Eq. (3.13) by

$$W_{\text{break}} = R[S(T_1 + T_2 + T_3) + T_1 T_2 + T_2 T_3 + T_3 T_1] + iM(LS + T_1 T_2 T_3). \quad (3.16)$$

The terms proportional to ST_A come from the geometrical fluxes, those proportional to $T_A T_B$ are generated by the R-R two-form fluxes F_2 , the $T_1 T_2 T_3$ term by the zero-form flux F_0 and the term linear to S originates from the NS three-form fluxes H_3 . The minimization conditions now give: $S = T_A = T = \sqrt{L}$, $U_3 = U + w(S)/2$. The supersymmetry is broken and the potential is again (locally) semi-positive-definite, compatible with flat space-time. The goldstino superfield is a combination of U_1, U_2 and U_3 superfields. In this example as well, the S -direction does not break supersymmetry.

Similar situations are met in type IIB. Depending on the kind of confining gauge group (D_3 -branes versus D_7 -branes) the gauge coupling constant can be either S or one of the T_A moduli, say T_1 . Let us consider for instance the following superpotential, valid in the presence of D_3 -branes:

$$W_{\text{total}} = A(U_1 - U_2)(S - U_3 - w(S)) + W_{\text{break}} \quad (3.17)$$

with

$$W_{\text{break}} = iB(LS + U_1 U_2 U_3). \quad (3.18)$$

The terms proportional to S originate from H_3 ; the others from F_3 .

In the absence of W_{break} , supersymmetry remains unbroken even when NS-NS and R-R three-form fluxes are on, as in example (3.17). With the actual W_{break} [Eq. (3.18)], supersymmetry is broken with semi-positive-definite potential as in all previous examples. The minimization of the potential leads to:

$$U_1 = U_2 = U, \quad S = U_3 + w(S), \quad LS = U^2[S - w(S)], \quad (3.19)$$

where we assumed U and S real. The goldstino superfield is defined by the T_A 's, which cancel the negative contribution of the potential and give rise to a no-scale structure, even in the presence of nonperturbative terms. Finally

$$W_{\text{min}} = 2iBU^2[S - w(S)] = 2iBLS. \quad (3.20)$$

We can examine the other case, namely when the gauge coupling constant is T_1 . We must then replace $w(S)$ by $w(T_1)$,

$$W_{\text{total}} = A(U_1 - U_2)(S - U_3 - w(T_1)) + W_{\text{break}}, \quad (3.21)$$

and choose the same (or a different) breaking term. Choosing W_{break} as in Eq. (3.18), the minimization of the potential leads to:

$$U_1 = U_2 = U, \quad S = U_3 + w(T_1), \quad LS = U^2[S - w(T_1)]. \quad (3.22)$$

The goldstino superfield is defined again by the T_A 's and the potential has a no-scale structure as before, even in the presence of the T_1 -dependent nonperturbative terms.

Notice incidentally that μ^3 in $w(S \text{ or } T_A)$ [Eq. (2.24)] can generally depend on Z_i moduli. This dependence has no consequence on the various properties discussed previously.

In the examples we have exhibited so far, the role of the gaugino condensate was important but not crucial for the supersymmetry breaking. An explicit breaking term, generated by a specific combination of fluxes, had to be superimposed to the mass terms, in order for the supersymmetry to be broken, independently of the presence of the condensate.

This situation is not generic, and we will now describe another class of examples, where supersymmetry breaking is triggered by the gaugino condensate.

3.3 Gaugino-induced supersymmetry breaking in type IIA

One generic feature of the breaking mechanism described in Sec. 3.2 is that the nonperturbative contribution to the gravitino mass is negligible. On the contrary, we will now analyze situations where the gaugino condensate breaks supersymmetry, and $m_{3/2}$ turns out to be related to the gaugino scale $w(S)$ only.

The generic form of the superpotential is again

$$W = W_{\text{susy}} + \mu^3(Z_i) e^{-S}. \quad (3.23)$$

The nonperturbative term can be used to induce supersymmetry breaking and thus it fully contributes to the effective theory at the vacuum. In other words, in contrast to previous examples, $\mu^3(Z_i)$ will not vanish at the minimum.

In order to fix the ideas we will examine this situation in detail in examples of type IIA orientifolds. The structure of those examples shows how generic is the procedure in the present framework.

Consider the superpotential

$$W = (T_1 - T_2)(-U_1 + U_2 - T_3 + 2S) + (U_1 T_3 - L) w(S), \quad (3.24)$$

which is generated by geometric and F_2 fluxes. It falls in the class mentioned in Sec. 3.1, where there is a partition between directions which break supersymmetry (here T_1 and T_2) and directions which preserve supersymmetry (T_3, U_1, U_2 and S).

The requirement $\langle W_{T_1} \rangle = \langle W_{T_2} \rangle = 0$ ensures $\langle V \rangle = 0$ since W is independent of U_3 , and the resulting supersymmetry-breaking condition reads

$$-U_1 + U_2 - T_3 + 2S = 0. \quad (3.25)$$

The vanishing of the F -auxiliary fields in the directions T_3, U_1, U_2 and S leads to the following equations:

$$\xi (\overline{U}_1 + U_2 - T_3 + 2S) - (\overline{U}_1 T_3 + L) w(S) = 0, \quad (3.26)$$

$$\xi (-U_1 - \overline{U}_2 - T_3 + 2S) + (U_1 T_3 - L) w(S) = 0, \quad (3.27)$$

$$\xi (-U_1 + U_2 + \overline{T}_3 + 2S) - (U_1 \overline{T}_3 + L) w(S) = 0, \quad (3.28)$$

$$\xi (-U_1 + U_2 - T_3 - 2\overline{S}) + (U_1 T_3 - L) (1 + S + \overline{S}) w(S) = 0, \quad (3.29)$$

where we have introduced

$$\xi \equiv T_1 - T_2. \quad (3.30)$$

The minimization condition (3.7) reads here

$$\text{Re } \xi = 0. \quad (3.31)$$

Equations (3.25)–(3.31) must be solved for ξ, T_3, U_1, U_2 and S . Combining Eqs. (3.26) and (3.28), one concludes that $T_3 = U_1$. A similar combination of Eqs. (3.26) and (3.27) shows that these moduli must be chosen real: $T_3 = U_1 = t$. The requirement (3.31) can be fulfilled by adjusting appropriately the imaginary part of the S field: $S = s - i\frac{\pi}{2} + 3i\varphi_\mu$ ($\mu = |\mu| \exp i\varphi_\mu$). This implies through Eq. (3.25) that $U_2 = u + i\pi - 6i\varphi_\mu$.

Finally, the equations for t, u and s are (3.25), a combination of (3.26) and (3.27) as well as a combination of (3.25), (3.26) and (3.29), which read:

$$u + 2(s - t) = 0, \quad (3.32)$$

$$t(t^2 - L) - u(t^2 + L) = 0, \quad (3.33)$$

$$t^5 + 2Lt^3 - 4Lt^2 - 3L^2t - 4L^2 = 0. \quad (3.34)$$

Furthermore ξ will be given by any of the equations (3.26) to (3.29), once the other moduli are fixed:

$$\xi = \frac{t^2 + L}{u + 2s} w. \quad (3.35)$$

Similarly, the gravitino mass will read:

$$e^{-K/2} m_{3/2} = \langle W \rangle = (t^2 - L) w(s). \quad (3.36)$$

From Eq. (3.34), assuming $L > 0$ we obtain:

$$L = t^2 \frac{t - 2 + 2\sqrt{t^2 + 1}}{3t + 4}; \quad (3.37)$$

the choice for the sign of the square root is the only one compatible with positivity of all moduli. Assuming L large, the leading and sub-leading behaviour for t is

$$t = \sqrt{L} + 1 + \mathcal{O}\left(\frac{1}{\sqrt{L}}\right). \quad (3.38)$$

Equation (3.33) gives thus

$$u = 1 + \mathcal{O}\left(\frac{1}{L}\right), \quad (3.39)$$

whereas from (3.32) we get:

$$s = \sqrt{L} + \frac{1}{2} + \mathcal{O}\left(\frac{1}{\sqrt{L}}\right). \quad (3.40)$$

As advertised previously, supersymmetry breaking is measured by

$$\text{Im } \xi \approx \sqrt{L} |\mu|^3 e^{-\sqrt{L}} \quad (3.41)$$

and the gravitino mass scales as

$$e^{-K/2} m_{3/2} \approx 2i\sqrt{L} |\mu|^3 e^{-\sqrt{L}}. \quad (3.42)$$

In the example at hand, the gaugino condensate is entirely responsible for the breaking of supersymmetry, as shown by the last two equations. Furthermore, the fluxes generating the superpotential (3.24) are not fine-tuned, and solutions for the moduli exist generically.

In fact, many examples of the above type can be found in type IIA, which are generated by fluxes originating from string theory compactifications. Consistent superpotentials with similar gaugino-condensation-induced supersymmetry breaking also exist, which are valid as supergravities without stringy origin. This happens for example with

$$W = (T_1 - T_2) (U_1 + U_2 + bU_3 + gU_1U_2 (U_3 - i\pi) - 2bS) + (U_1U_2 - L) w(S), \quad (3.43)$$

where the cubic term cannot be obtained by switching on fluxes in type IIA string compactifications. Although this superpotential turns out to possess the various features advertised so far, we will not further elaborate on it.

3.4 Gaugino-induced supersymmetry breaking in heterotic

Although type II and heterotic compactifications are quite similar when supersymmetry breaking is mostly due to fluxes [see Sec. (3.2)], they turn out to be drastically different in cases where the breaking of supersymmetry is induced by a gaugino condensate. The reason boils down to the absence of S contributions to the superpotential, directly originating from fluxes.

Consider for concreteness a superpotential of the type

$$W = \hat{A}U_1 + \hat{B}U_2 + \hat{C}U_3 + \hat{D}U_4, \quad (3.44)$$

where $U_4 = U_1 U_1 U_3$. This superpotential is odd in the U_i 's and captures most of the heterotic compactifications considered in the present work, with a gaugino condensate. We have introduced the following functions of T_1, T_2 and S :

$$\hat{A} = [\alpha + \alpha' w(S)] \xi + A w(S), \quad (3.45)$$

$$\hat{B} = [\beta + \beta' w(S)] \xi + B w(S), \quad (3.46)$$

$$\hat{C} = [\gamma + \gamma' w(S)] \xi + C w(S), \quad (3.47)$$

$$\hat{D} = [\delta + \delta' w(S)] \xi + D w(S), \quad (3.48)$$

where $\xi = T_1 - T_2$ as defined in (3.30) and $w(S)$ in (2.24).

The minimization condition (3.7) reads $\text{Re } \xi = 0$, as in the examples of type IIA (Sec. 3.3). We will therefore choose $S = s - i\pi/2 + 3i\varphi_\mu$ and $U_i = u_i$ real. Everything is consistent provided $\alpha, \beta, \gamma, \delta$ and A, B, C, D are real and $\alpha', \beta', \gamma', \delta'$ are imaginary.

The no-scale requirement $\langle V \rangle = 0$ is fulfilled provided $\langle W_{T_1} \rangle = \langle W_{T_2} \rangle = 0$ (W is independent of T_3). The corresponding condition reads:

$$(\alpha + \alpha' w)u_1 + (\beta + \beta' w)u_2 + (\gamma + \gamma' w)u_3 + (\delta + \delta' w)u_4 = 0, \quad (3.49)$$

whereas the vanishing of the U_A -auxiliary fields leads to

$$-\hat{A}u_1 + \hat{B}u_2 + \hat{C}u_3 - \hat{D}u_4 = 0, \quad (3.50)$$

$$\hat{A}u_1 - \hat{B}u_2 + \hat{C}u_3 - \hat{D}u_4 = 0, \quad (3.51)$$

$$\hat{A}u_1 + \hat{B}u_2 - \hat{C}u_3 - \hat{D}u_4 = 0. \quad (3.52)$$

These equations can be solved. They indeed imply that

$$\hat{A}u_1 = \hat{B}u_2 = \hat{C}u_3 = \hat{D}u_1 u_2 u_3, \quad (3.53)$$

which allows therefore to express u_1, u_2, u_3 in terms of ξ, s :

$$u_1 = \sqrt{\frac{\hat{B}\hat{C}}{\hat{A}\hat{D}}}, \quad u_2 = \sqrt{\frac{\hat{A}\hat{C}}{\hat{B}\hat{D}}}, \quad u_3 = \sqrt{\frac{\hat{A}\hat{B}}{\hat{C}\hat{D}}}, \quad u_4 = \sqrt{\frac{\hat{A}\hat{B}\hat{C}}{\hat{D}^3}}. \quad (3.54)$$

The fields ξ and s are in turn determined by Eq. (3.49) and the equation for the S -auxiliary field:

$$W + 2s[(\alpha'\xi + A)u_1 + (\beta'\xi + B)u_2 + (\gamma'\xi + C)u_3 + (\delta'\xi + D)u_4]w = 0, \quad (3.55)$$

which can be further simplified by using Eqs. (3.49) and (3.54):

$$\frac{2}{s} = -4 - \left(\frac{\alpha'}{\hat{A}} + \frac{\beta'}{\hat{B}} + \frac{\gamma'}{\hat{C}} + \frac{\delta'}{\hat{D}} \right) \xi w. \quad (3.56)$$

In order to solve for ξ and s , we will introduce a set of intermediate imaginary quantities λ_i , defined by

$$\hat{A} = \lambda_1 \xi w, \quad \hat{B} = \lambda_2 \xi w, \quad \hat{C} = \lambda_3 \xi w, \quad \hat{D} = \lambda_4 \xi w. \quad (3.57)$$

Using Eqs. (3.45)–(3.48), these parameters are expressed in terms of ξ and s :

$$\alpha \xi + A w + (\alpha' - \lambda_1) \xi w = 0, \quad (3.58)$$

$$\beta \xi + B w + (\beta' - \lambda_2) \xi w = 0, \quad (3.59)$$

$$\gamma \xi + C w + (\gamma' - \lambda_3) \xi w = 0, \quad (3.60)$$

$$\delta \xi + D w + (\delta' - \lambda_4) \xi w = 0. \quad (3.61)$$

The λ_i 's allow to express all u_i 's as

$$u_1 = \sqrt{\frac{\lambda_2 \lambda_3}{\lambda_1 \lambda_4}}, \quad u_2 = \sqrt{\frac{\lambda_1 \lambda_3}{\lambda_2 \lambda_4}}, \quad u_3 = \sqrt{\frac{\lambda_1 \lambda_2}{\lambda_3 \lambda_4}}, \quad u_4 = \sqrt{\frac{\lambda_1 \lambda_2 \lambda_3}{\lambda_4^3}}. \quad (3.62)$$

These expressions show in particular that an even number of $i\lambda_i$'s can be negative. Finally, Eqs. (3.49) and (3.56) read:

$$\frac{\alpha + \alpha' w}{\lambda_1} + \frac{\beta + \beta' w}{\lambda_2} + \frac{\gamma + \gamma' w}{\lambda_3} + \frac{\delta + \delta' w}{\lambda_4} = 0 \quad (3.63)$$

and

$$\frac{2}{s} = -4 - \left(\frac{\alpha'}{\lambda_1} + \frac{\beta'}{\lambda_2} + \frac{\gamma'}{\lambda_3} + \frac{\delta'}{\lambda_4} \right). \quad (3.64)$$

These are the final equations for determining ξ and s . We will now proceed and show that solutions with large and positive s , together with exponentially small ξ (*i.e.* perturbative regime and small supersymmetry breaking) do indeed exist under minor and natural assumptions on the fluxes (coefficients $\alpha, \beta, \gamma, \delta, \alpha', \beta', \gamma', \delta'$ and A, B, C, D).

For simplicity, we restrict ourselves to the plane-symmetric situation, where

$$\alpha = \beta = \gamma, \quad \alpha' = \beta' = \gamma', \quad A = B = C, \quad (3.65)$$

which imply that

$$\lambda_1 = \lambda_2 = \lambda_3 \equiv \lambda \quad \text{and} \quad u_1 = u_2 = u_3 = \sqrt{\frac{\lambda}{\lambda_4}} \equiv u. \quad (3.66)$$

The set of equations we must solve is therefore (3.63) and (3.64) together with those defining λ and λ_4 , namely (3.58) and (3.61). We can eliminate ξ through the latter equations and use (3.63) to express λ and λ_4 as functions of s only:

$$\frac{1}{\lambda} = \frac{w}{3(\alpha + \alpha'w)} \frac{3D\alpha + A\delta + (3\alpha'D + A\delta')w}{D\alpha - A\delta + (D\alpha' - A\delta')w}, \quad (3.67)$$

$$\frac{1}{\lambda_4} = -\frac{w}{(\delta + \delta'w)} \frac{3D\alpha + A\delta + (3\alpha'D + A\delta')w}{D\alpha - A\delta + (D\alpha' - A\delta')w}. \quad (3.68)$$

We can now express $\xi = \xi(s)$ using either (3.58) or (3.61) together with (3.67) and (3.68):

$$4\xi = -\frac{3D\alpha + A\delta + (3D\alpha' + A\delta')w}{(\alpha + \alpha'w)(\delta + \delta'w)} w, \quad (3.69)$$

whereas the central equation for the determination of s will be given by (3.64):

$$\frac{2}{s} = -4 - \frac{(\alpha'\delta - \delta'\alpha)w}{(\alpha + \alpha'w)(\delta + \delta'w)} \frac{3D\alpha + A\delta + (3\alpha'D + A\delta')w}{D\alpha - A\delta + (D\alpha' - A\delta')w}. \quad (3.70)$$

We would like now to show that Eq. (3.70) admits physically acceptable solutions for s , provided that the fluxes $\alpha, \delta, \alpha', \delta', A, D$ are large while their ratios are of order unity. If this requirement is fulfilled we can define a variable ρ (real function of s) as

$$\rho = i \frac{D\alpha - A\delta}{D\alpha w}, \quad (3.71)$$

which can be consistently taken to be of order one since w is small and $D\alpha/A\delta$ of order one.

Under these assumptions, we can perform an expansion in powers of w for all quantities. We find the following dominant contributions:

$$\xi \approx -\frac{D}{\delta} w \quad (3.72)$$

and

$$\frac{1}{2s} \approx -1 - \frac{D}{\delta} \frac{\alpha'\delta - \delta'\alpha}{D\alpha' - A\delta' - iD\alpha\rho}. \quad (3.73)$$

For further simplification we specialize to

$$\alpha' = i\alpha, \quad \delta' = -i\delta. \quad (3.74)$$

Equations (3.67) and (3.68) give, at dominant order,

$$\frac{1}{2s} \approx \frac{4 - \rho}{\rho - 2} \iff \rho \approx 2 \frac{4s + 1}{2s + 1}. \quad (3.75)$$

Notice also that from Eq. (3.66) we obtain

$$u \approx \sqrt{\frac{-3\alpha}{\delta}}. \quad (3.76)$$

Inserting the definition of ρ [Eq. (3.71)] into (3.75) we obtain the following equation for s , at dominant order:

$$\frac{A\delta - D\alpha}{D\alpha} \approx 2i \frac{4s+1}{2s+1} w(s). \quad (3.77)$$

As advertised previously, this equation is compatible with large values of s . In that regime it further simplifies:

$$s \approx \log \left(\frac{4|\mu|^3 D\alpha}{D\alpha - A\delta} \right) - \frac{1}{4 \log \left(\frac{4|\mu|^3 D\alpha}{D\alpha - A\delta} \right)} \quad (3.78)$$

We can finally determine the gravitino mass. Using Eqs. (3.44), (3.57) and (3.62), we obtain:

$$\begin{aligned} e^{-K/2} m_{3/2} &= \langle W \rangle \\ &= 4u^3 \lambda_4 \xi w = 4 \sqrt{\frac{\lambda^3}{\lambda_4}} \xi w \\ &= -\frac{A\delta' - D\alpha' + \frac{A\delta - D\alpha}{w}}{\alpha + \alpha' w} \left(-3 \frac{\alpha + \alpha' w}{\delta + \delta' w} \right)^{3/2} w^2 \end{aligned} \quad (3.79)$$

for generic plane-symmetric situations. In the special case captured by (3.74) and within the above approximations, the result is

$$e^{-K/2} m_{3/2} \approx i4D \left(-\frac{3\alpha}{\delta} \right)^{3/2} \frac{s}{2s+1} w^2, \quad (3.80)$$

with s given in Eq. (3.78). The gravitino mass scales as w^2 instead of w like in type IIA (Sec. 3.3). This is due to the absence of flux-induced S -term in the heterotic superpotential.

4 Conclusions

Understanding the effect of nonperturbative corrections in the presence of fluxes is an important issue. As already stressed, this can shed light on the nature of the vacuum and the stabilization of moduli, and turns out to be a valuable tool for circumventing the runaway behaviour of moduli or the fine-tuning problem.

In the present paper, we have addressed these questions through a selection of examples, using the effective supergravity analysis as obtained from type II orientifolds and heterotic

string compactifications. The important outcome of this analysis is that the pathological behaviour of the vacuum in the presence of fluxes with nonperturbative corrections *is not* a generic property of the $N = 1$ effective supergravity, with or without spontaneously broken supersymmetry: the usual caveats quoted previously can be avoided with a suitable combination of flux and nonperturbative contributions.

We would like to emphasize the importance of analyzing the entire superpotential, including both flux-induced perturbative and nonperturbative contributions. Although nonperturbative corrections do not necessarily trigger supersymmetry breaking, they can alter various terms and must therefore be taken into account at a very early stage. Most importantly, they can severely change the picture of moduli stabilization drawn by fluxes only. This has been illustrated in several examples in Sec. 3.

The examples that we have investigated enable us to conclude that at least *three* scaling behaviours of the gravitino mass exist:

1. In models where the nonperturbative corrections do not trigger the supersymmetry breaking – they modify the mass terms though – the gravitino mass scales like:

$$m_{3/2} = c / \sqrt{V},$$

where $V = \exp K$ is the volume of the moduli space and c is related to the flux numbers. In this case, any mass hierarchy strongly relies on the volume V .

2. In type II models where the nonperturbative contributions to the superpotential induce the supersymmetry breaking, the gravitino mass is controlled by the nonperturbative superpotential $w(S)$. We have given examples where

$$m_{3/2} = c w(S) / \sqrt{V}, \tag{4.1}$$

as commonly expected from nonperturbative supersymmetry breaking. In this case, a mass hierarchy can be created irrespectively of the size of the volume of the moduli space.

3. A *third* scaling behaviour actually appears in the framework of heterotic string when supersymmetry breaking is directly induced by the gaugino condensate. There, the gravitino mass is more unusual but still generic:

$$m_{3/2} = c w(S)^2 / \sqrt{V}. \tag{4.2}$$

This creates a mass hierarchy stronger than in type IIA [Eq. (4.1)]. The heterotic realizations under consideration are actually quite generic despite the fact that the ratios between flux coefficients are required to be of order one, while the coefficients themselves are large.

The last case shows that the analysis of heterotic models with supersymmetry breaking induced by non-perturbative effects deserves a more systematic investigation. To be complete, the latter might necessitate the inclusion of glueball terms of the type $\langle FF \rangle$ in the superpotential, together with the gaugino $\langle \lambda\lambda \rangle$ ones. It should shed light on the validity of various gaugino-condensate-induced supersymmetry breaking scenarios advertised in the literature but not captured by our systematic analysis.

One should finally stress that the analysis of the $N = 1$, low-energy soft-supersymmetry-breaking terms strongly depends on the class of model under consideration since their pattern is mostly controlled by the radiative corrections induced by the supersymmetry-breaking sector.

Acknowledgements

We thank Fabio Zwirner for discussions. J.-P. D. thanks the Ecole Normale Supérieure and Ecole Polytechnique. C. K. and P. M. P. thank the Université de Neuchâtel for hospitality. This work was supported in part by the EU under the contracts MEXT-CT-2003-509661, MRTN-CT-2004-005104, MRTN-CT-2004-503369, by the Agence Nationale pour la Recherche, France, contract 05-BLAN-0079-01, and by the Swiss National Science Foundation.

References

- [1] S. Ferrara, L. Girardello and H. P. Nilles, Phys. Lett. B **125** (1983) 457.
- [2] J.-P. Derendinger, L. E. Ibáñez and H. P. Nilles, Phys. Lett. B **155** (1985) 65;
M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B **156** (1985) 55;
C. Kounnas and M. Porrati, Phys. Lett. B **191** (1987) 91.
- [3] E. Witten, Phys. Lett. B **155** (1985) 151;
J.-P. Derendinger, L. E. Ibáñez and H. P. Nilles, Nucl. Phys. B **267** (1986) 365.
- [4] D. Lüst, S. Reffert and S. Stieberger, Nucl. Phys. B **706** (2005) 3
[arXiv:hep-th/0406092].
- [5] J.-P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, Nucl. Phys. B **715** (2005) 211 [arXiv:hep-th/0411276] and Fortsch. Phys. **53** (2005) 926
[arXiv:hep-th/0503229].
- [6] J.-P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, to appear.
- [7] A. Strominger, Nucl. Phys. B **274** (1986) 253;
R. Rohm and E. Witten, Annals Phys. **170** (1986) 454;
N. Kaloper and R. C. Myers, JHEP **9905** (1999) 010 [arXiv:hep-th/9901045].
- [8] J. Polchinski and A. Strominger, Phys. Lett. B **388** (1996) 736 [arXiv:hep-th/9510227];
I. Antoniadis, E. Gava, K. S. Narain and T. R. Taylor, Nucl. Phys. B **511** (1998) 611
[arXiv:hep-th/9708075];
S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B **584** (2002) 69 [Erratum-ibid. B **608**
(2001) 477] [arXiv:hep-th/9906070];
S. Gukov, Nucl. Phys. B **574** (2000) 169 [arXiv:hep-th/9911011].
- [9] J. Michelson, Nucl. Phys. B **495** (1997) 127 [arXiv:hep-th/9610151];
K. Dasgupta, G. Rajesh and S. Sethi, JHEP **9908** (1999) 023 [arXiv:hep-th/9908088];
T. R. Taylor and C. Vafa, Phys. Lett. B **474** (2000) 130 [arXiv:hep-th/9912152];
P. Mayr, Nucl. Phys. B **593** (2001) 99 [arXiv:hep-th/0003198];
G. Curio, A. Klemm, D. Lüst and S. Theisen, Nucl. Phys. B **609** (2001) 3

- [arXiv:hep-th/0012213];
S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66** (2002) 106006 [arXiv:hep-th/0105097];
S. Kachru, M. B. Schulz and S. Trivedi, JHEP **0310** (2003) 007 [arXiv:hep-th/0201028];
A. R. Frey and J. Polchinski, Phys. Rev. D **65** (2002) 126009 [arXiv:hep-th/0201029];
S. Ferrara and M. Porrati, Phys. Lett. B **545** (2002) 411 [arXiv:hep-th/0207135];
C. Angelantonj, S. Ferrara and M. Trigiante, JHEP **0310** (2003) 015 [arXiv:hep-th/0306185] and Phys. Lett. B **582** (2004) 263 [arXiv:hep-th/0310136];
M. Grana, T. W. Grimm, H. Jockers and J. Louis, Nucl. Phys. B **690**, 21 (2004) [arXiv:hep-th/0312232];
T. W. Grimm and J. Louis, Nucl. Phys. B **699**, 387 (2004) [arXiv:hep-th/0403067];
P. G. Camara, L. E. Ibáñez and A. M. Uranga, Nucl. Phys. B **708** (2005) 268 [arXiv:hep-th/0408036];
H. Jockers and J. Louis, Nucl. Phys. B **705**, 167 (2005) [arXiv:hep-th/0409098].
- [10] M. Grana, R. Minasian, M. Petrini and A. Tomasiello, JHEP **0408** (2004) 046 [arXiv:hep-th/0406137];
C. M. Hull and R. A. Reid-Edwards, arXiv:hep-th/0503114;
T. House and E. Palti, Phys. Rev. D **72** (2005) 026004 [arXiv:hep-th/0505177];
M. Grana, J. Louis and D. Waldram, arXiv:hep-th/0505264.
- [11] R. Rohm, Nucl. Phys. B **237** (1984) 553;
C. Kounnas and M. Porrati, Nucl. Phys. B **310** (1988) 355;
S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Nucl. Phys. B **318** (1989) 75;
M. Porrati and F. Zwirner, Nucl. Phys. B **326** (1989) 162;
C. Kounnas and B. Rostand, Nucl. Phys. B **341** (1990) 641;
I. Antoniadis, Phys. Lett. B **246** (1990) 377;
I. Antoniadis and C. Kounnas, Phys. Lett. B **261** (1991) 369;
E. Kiritsis and C. Kounnas, Nucl. Phys. B **503** (1997) 117 [arXiv:hep-th/9703059];
I. Antoniadis, E. Dudas and A. Sagnotti, Nucl. Phys. B **544** (1999) 469 [arXiv:hep-th/9807011] and Phys. Lett. B **464** (1999) 38 [arXiv:hep-th/9908023];
I. Antoniadis, J.-P. Derendinger and C. Kounnas, Nucl. Phys. B **551** (1999) 41 [arXiv:hep-th/9902032], and arXiv:hep-th/9908137.

- [12] A. Das, Phys. Rev. D **15** (1977) 2805;
 E. Cremmer and J. Scherk, Nucl. Phys. B **127** (1977) 259;
 E. Cremmer, J. Scherk and S. Ferrara, Phys. Lett. B **74** (1978) 61;
 A. H. Chamseddine, Nucl. Phys. B **185** (1981) 403;
 J.-P. Derendinger and S. Ferrara, *Lectures given at Spring School of Supergravity and Supersymmetry, Trieste, Italy, Apr 4-14, 1984*, CERN-TH-3903;
 M. de Roo, Nucl. Phys. B **255** (1985) 515 and Phys. Lett. B **156** (1985) 331;
 E. Bergshoeff, I. G. Koh and E. Sezgin, Phys. Lett. B **155** (1985) 71;
 M. de Roo and P. Wagemans, Nucl. Phys. B **262** (1985) 644 and Phys. Lett. B **177** (1986) 352;
 P. Wagemans, Phys. Lett. B **206** (1988) 241 and *Aspects of $N=4$ supergravity*, Ph.D. Thesis, Groningen University report RX-1299 (1990).
- [13] G.L. Cardoso, G. Curio, G. Dall'Agata, D. Lüst, Fortsch. Phys. **52** (2004) 483 [arXiv:hep-th/0310021];
 G.L. Cardoso, G. Curio, G. Dall'Agata, D. Lüst, JHEP **0409** (2004) 059 [arXiv:hep-th/0406118];
 L. Gorlich, S. Kachru, P. K. Tripathy and S. P. Trivedi, JHEP **0412** (2004) 074 [arXiv:hep-th/0407130];
 K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP **0411** (2004) 076 [arXiv:hep-th/0411066];
 P. Manousselis, N. Prezas and G. Zoupanos, arXiv:hep-th/0511122.
- [14] G. Dall'Agata and S. Ferrara, Nucl. Phys. B **717**, 223 (2005) [arXiv:hep-th/0502066];
 L. Andrianopoli, M. A. Lledo and M. Trigiante, JHEP **0505**, 051 (2005) [arXiv:hep-th/0502083];
 G. Dall'Agata, R. D'Auria and S. Ferrara, Phys. Lett. B **619**, 149 (2005) [arXiv:hep-th/0503122];
 G. Dall'Agata and N. Prezas, JHEP **0510** (2005) 103 [arXiv:hep-th/0509052].
- [15] L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B **355**, 649 (1991);
 J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B **372**, 145 (1992) and Phys. Lett. B **271**, 307 (1991).

- [16] E. Kiritsis, C. Kounnas, P. M. Petropoulos and J. Rizos, Phys. Lett. B **385** (1996) 87 [arXiv:hep-th/9606087] and Nucl. Phys. B **540** (1999) 87 [arXiv:hep-th/9807067].
- [17] G. Veneziano and S. Yankielowicz, Phys. Lett. B **113** (1982) 231.
- [18] K. Konishi, Phys. Lett. B **135** (1984) 439.
- [19] G. Villadoro and F. Zwirner, hep-th/0508167.
- [20] F. Buccella, J.-P. Derendinger, S. Ferrara and C. A. Savoy, Phys. Lett. B **115** (1982) 375;
C. Procesi and G. W. Schwarz, Phys. Lett. B **161**, 117 (1985).
- [21] J.-P. Derendinger and A. Zaugg, in preparation.
- [22] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133** (1983) 61;
J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **134** (1984) 429;
J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **247** (1984) 373;
A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145** (1987) 1.